The effect of urban morphology on the solar capacity of three-dimensional cities

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A B S T R A C T

As a clean and renewable resource, solar energy is increasingly being used to relieve the pressures on environmental protection and the exhaustion of conventional energy. Although photovoltaic modules have been installed in many cities, the lack of quantitative mapping of the annual solar energy potential of urban surfaces hinders the effective utilization of solar energy. Herein, we provide a solar irradiation estimation solution for three-dimensional (3D) cities to quantify annual irradiations on urban envelopes and to investigate the effect of urban morphology on the resulting solar capacity. By modelling urban surfaces as 3D point clouds, annual irradiances of the point clouds were estimated. An empirical investigation across ten cities suggests that urban areas at lower latitudes tend to have larger values of annual irradiation; moreover, an area having greater building heights consistently has the largest third quartile of irradiation compared with lower buildings in the same city. Conversely, areas with many low buildings have a larger proportion of useable areas; in this arrangement, façades can optimally utilize solar energy, meaning that large irradiations are concentrated on certain façades. The Pearson correlation coefficients between solar capacity and urban morphology indices suggest that urban morphology has an important effect on solar capacity.

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1. Introduction

Presently, pressures from the exhaustion of conventional energy (e.g., fossil fuels) and environmental pollution promote wider adoption of renewable energies [1]. As a readily available source of renewable energy with a large magnitude (630,000 EJ/yr on the land surface) [2], solar energy is increasingly becoming an appealing source of electricity worldwide [3,4]. Photovoltaic (PV) modules are usually installed on rooftops to reduce reliance on the public electricity network [5]. In addition, semitransparent PV modules have provided an alternative substitute to conventional window glass [6–10], generating considerable amounts of electricity and allowing less solar radiation to penetrate indoor spaces, thus eventually reducing energy needs for air conditioning. This benefit would be substantial for cities where air conditioners are intensively used over almost the entire year, such as Hong Kong and Singapore. Additionally, photovoltaic roads have been recently built [11,12]. These applications imply excellent potential for utilizing solar energy in cities. Therefore, mapping the distribution of annual solar irradiances and understanding the effect of urban morphology on solar capacity is imperative for the systematic and practical use of solar energy.

In order to investigate the effect of urban morphology, it is vital to provide an accurate estimation of solar energy in an urban environment. Many studies estimated photovoltaic potential on rooftops [13–17], in which a study estimated solar photovoltaic potential on rooftops in Hong Kong by incorporating a cloud...
probability map [16]. Since cloud cover can determine solar energy on urban envelopes, possibly with seasonal variation, the study will also use historical data on cloud cover to determine direct solar irradiation from the atmosphere.

Other studies in estimating solar energy mainly focused on façades associated with ground [18] or rooftops [19–22]. For instance, one study proposed an algorithm to generate hyper points on façades so that each point enriches with solar irradiation [19]. In the same strategy, this study will model a city as three-dimensional (3D) point clouds covering the whole urban envelopes to present annual solar irradiation on the clouds. The other two studies mainly contributed to the visualization of solar irradiation [21,22], which incorporated a r.sun model and achieved real-time rendering by applying the technology on graphics processing unit acceleration. However, it has not considered a scenario that neighborhood buildings may interrupt solar irradiation significantly, which is non-negligible, particularly in megacities where skyscrapers clusters. Thus, overcoming this challenge becomes one of the research objectives. Another study emphasized the effect of greeneries on receiving solar irradiation [23], which, however, will not be considered in this study because buildings are the most critical entity that fundamentally determines urban morphology in cities.

In summary, this study's objective is to visualize solar irradiation on all three partitions, including rooftops, façades, and ground [23–27]. In particular, one study utilized the r.sun model to compute the segmentation between 3D vectors and the polygons and hence estimated shadows created by surrounding buildings [25]. Two similar studies examined whether a 3D vector intersects with a voxel to calculate solar irradiations on building surfaces [26,27]. However, it is uncertain about applying these models to buildings with complex geometries. For example, two façades respectively belonging to two buildings may touch with each other so that the lower façade will never make shadows, footprints of buildings may be concave polygons so that shadows will project on itself, and the footprints may nest with each other that make irregular shadows. Since cities are likely having a variety of layouts and structures, and creating a reliable 3D solar city is crucial to investigate the urban morphology effect, addressing all these scenarios becomes crucial in this study. This study will also make 3D intersections between all the radiation vectors and all the buildings containing different geometrics to construct shadows even made by far-away buildings, which thus can be applied for cities having a high density of tall buildings.

Different from rural areas, buildings are significant entities in urban areas that may equip with a large area of glass or mirrors with high albedo so that reflective radiation may alter the solar distribution significantly, making solar accumulation or dispersion in specific locations. A few studies indicated the capability of incorporating reflective radiations. For example, r.sun included the reflected component on horizontal or inclined surfaces [28,29]. SOLENE allowed a multi-reflection process in the indoor architecture space [30]. Daysim computed irradiation accounting for all the light, direct and inter-reflected from the sky patch [31], and RADIANCE supported reflected radiation on buildings [32–34]. However, the method of managing accumulative and dispersive processes of the reflective radiation between urban envelopes is unclear, and it is uncertain for the capability of these models to compute large urban areas with numerous buildings. The two deficiencies motivate us to propose a reflective module for estimating annual solar irradiation. It is vital because the albedo of urban envelopes is comparatively higher than in rural areas, and reflective radiation may constitute a considerable proportion of global radiation, especially in some locations with irradiation accumulation or dispersion.

A few studies investigated the relationship between urban morphology and solar irradiation in urban areas. Some explored relationships between urban geometry and solar accessibility in different temporal scales, which focused on rooftops and façades in several hours of a day [27], orientations of tilted roofs and façades subject to seasonality [35], and a wide range of building densities in different seasons [18]. In comparison, this study will investigate the capacity of a city to maintain solar energy over an entire year, which is particularly important for practical implications such as the installation of PV modules. In another aspect, the effect of street characteristics on solar irradiation was explored. One proposed an indicator, which equals to the difference of the building heights divided by distance between buildings, to analysis its influence of solar radiation on façades [36], versus one investigated the relationship between different orientations of buildings and solar irradiation on tilted roofs and façades [35]. Instead of focusing on a single characteristic, the other two studies proposed several indices of urban morphology to explore their relationships between solar accessibility on building envelopes, and analyzed several scenarios of urban morphology and tested variables of urban form in order to increase solar energy potential for better urban design [37,38]. Comparing with the above studies that either relied on a simulated urban form or a single city, this study aims to obtain a knowledge of the effect of urban morphology through analyzing the solar capacity of several cities globally.

In summary, this study will make a four-fold contribution. First, it models 3D shadows made by all buildings, which is particularly essential for urban areas with high-rise and high-density buildings. Second, this work incorporates multiple reflections with direct solar radiations between the urban envelopes with various types of geometries, which includes not only rooftops and the ground but also façades in a variety of orientations. Third, it allows fast computation of large amounts of buildings. Most importantly, this study will reveal the effect of urban morphology on the solar capacity of 3D cities. The approach proposed herein also can be applied to urban heat island mitigation [39,40], building daylight planning [27], and urban mobility [41,42].

This paper is organized as follows. Section 2 defines the concept of solar capacity. Section 3 constructs a 3D solar city, which incorporates direct and diffuse irradiations from the atmosphere with multiple reflective irradiations between urban envelopes. Section 4 introduces data collection and validates the model. Section 5 explores the ability of ten cities across the world to utilize solar energy. Then, Section 6 investigates the distribution of quantitatively high and spatially concentrate solar irradiations in three partitions on rooftops, façades, and the ground. Furthermore, Section 7 investigates the effect of urban morphology on solar capacity through correlation analysis. Finally, Section 8 and Section 9 make a discussion and conclusion.

2. Construction of a 3D solar city

A strategy for constructing a 3D solar city is presenting the urban envelopes (i.e., rooftops, façades, and ground) as 3D point clouds, and each point has a thematic attribute, e.g., the total solar irradiation accumulated over a year. At an instant of time, point clouds under sunshine or shadows can be simply determined by investigating intersections between 3D polygons of shadows and 3D polygons of urban envelopes. More specifically, point clouds on rooftops and ground are under shadows if they are below a 3D shadow polygon but inside the polygon in the orthographic projection. Similarly, point clouds on façades are also under shadows if they are orthographically below a 3D shadow line, i.e., the intersection between 3D shadow polygons and 3D façade polygons. Alternatively, point clouds are under sunshine if they are above the shadow polygons or shadow lines. Hence, multiple reflections can be estimated on urban envelopes in Fig. 1, and abbreviations are presented in Table 1.
2.1. Definition of the solar capacity

Annual irradiation is defined as the solar irradiation that, starting from the incoming atmospheric irradiation, remains after the process of multiple reflections between urban envelopes. However, the confounding factor of different weather conditions for different cities may obstruct the detection of urban morphology on annual irradiations. For example, two cities with the exact same urban morphology and latitude would still have a big difference in total irradiations if one city is sunny while the other one is cloudy throughout a year. Thus, instead of investigating annual irradiations, the concept of solar capacity (\(\text{cap}\)) to describe the ability of different morphologies to utilize solar energy is proposed, which equals the total amount of annually maintained irradiation from the atmosphere in the same size of a flat area without any infrastructure (\(u_a\)), i.e., \(\text{cap} = \frac{u_a}{v}\). The notion of solar capacity is not influenced by specific weather conditions, as weather only determines the values of direct irradiation from the atmosphere so that routings of multiple reflections and proportion of the trapped solar irradiation in each reflection will be the same with a given urban morphology and albedo. In other words, it is based on the transformation of the values of annual irradiation to the percentage of the annual irradiation that remains after multiple reflections in a specific urban morphology. Thus, the solar capacity is an appropriate index that focusing on the capability of urban morphology to utilize solar energy independently of the specific weather conditions.

2.2. A city as 3D point clouds

In a 3D city, a building (\(b_i\)) has a rooftop (\(g_i\)) and a set of vertical façades (\(g_{x,i} = \{g_{1,i}, \ldots, g_{n_i}\}\)), all of which are recorded as 3D polygons. A rooftop \(g_i\) coincides with a façade \(g_{x,i}\) (\(x \in \{1, n\}\)) as a line segment \(s\) (Fig. 2a). The ground (\(g_{g,i}\)) can be discretized as spatially contiguous and homogeneous grid cells with a resolution of \(d\) so that points can be generated at the center of the grids. Hence, 3D point clouds on \(g_i\) and \(g_{x,i}\) can be obtained and denoted by \(P_{g,i}\) and \(P_{g_{x,i}}\), respectively. Similarly, \(g_{x,i}\) can be discretized as a set of vertical grid cells in the same \(d\) so that point clouds on \(g_{x,i}\) can be derived and represented as \(P_{g_{x,i}}\), where their heights increase gradually from 0 to the height of buildings with a constant interval of \(d\). Finally, point clouds on \(b_i\) can be organized as \(\{P_{g,i}, P_{g_{x,i}}\}\) (\(i \in \{1, k\}\)), and a 3D city can be denoted by \(\mathcal{P} = \{P, P_{g_{x,i}}, P_{g_{x,i}}\}\).

2.3. 3D shadow surfaces

Solar radiation that passes through the atmosphere from elevation \(e\) and azimuth \(z\) has solar irradiation \(u\) at an instant of time \(t\) and location \(l\), i.e., \(sr = (u, e, z, t)\). A shadow polygon \(g_i\) in parallel with \(sr\) will form when \(sr\) passes through the rooftop at the line segment \(s\), which is a 3D parallelogram that divides sunshine from a shadow. The initial shadow polygon needs three modifications to make shadows physically correct. First, if two façades of two buildings touch with each other at a line segment \(s\), then \(g_i\) belonging to the lower building will be removed since the lower façade will never make shadows (Fig. 2a). Second, \(g_i\) may pass through several façades of the same building as a set of 3D shadow lines since the rooftop \(g_i\) is a concave polygon; however, this cannot happen in reality. In this case, the 3D shadow lines will generate a set of false shadow polygons that are unified as a single one \(g_{gp}\). Hence, \(g_{gp}\) will be cut off from \(g_i\), which means that \(g_{gp} = (g_{gp} - g_{gp})\) replaces \(g_{gp}\) (Fig. 2b). Third, \(g_i\) created from a single building \(b_j\) may also be obstructed by façades and rooftops of the other building \(b_j\). In more detail, \(g_i\) may pass through façades and/or rooftops of \(b_j\) as a set of 3D shadow lines that also generate false shadow polygons, which will be unified as \(g_{gp}\). Thus, \(g_{gp}\) will be modified as \(g_{gp} = (g_{gp} - g_{gp})\) (Fig. 2c). On top of this, physically reasonable shadows in the study area can be derived as \(\mathcal{S} = \{g_{gp}\}\), and all the 3D shadow lines are denoted by \(\mathcal{S}\) (Fig. 3).

2.4. Point clouds in sunshine and shadows

At this stage, 3D point clouds in \(\mathcal{P}\) can be determined to be either in sunshine or in shadow. First of all, \(\mathcal{P}\) and \(\mathcal{S}\) are orthographically projected as \(\mathcal{P}'\) and \(\mathcal{S}'\), respectively. Point clouds in \(P_{g_{x,i}}\) and \(P_{g_{x,i}}\) are in shadow if their orthographic projections are located in \(\mathcal{S}'\). Similarly, point clouds in \(P_{g_{x,i}}\) are in shadow if they are below a 3D shadow line in \(\mathcal{S}\) and their orthographic projections, are on the corresponding shadow line of \(\mathcal{S}'\). Fig. 4a presents the result of this step showing that point clouds are in a binary situation, i.e., shadows and sunshine are in blue and yellow, respectively. Based on a solar radiation \(sr\), the solar irradiation of point clouds in the sunshine and on horizontal surfaces (i.e., rooftops and the ground)

![Fig. 1. Flowchart of constructing a 3D solar city presented by point clouds.](image-url)
is computed as $u_r = u_g = u \sin(e)$ according to the energy conservation law. In contrast, the solar irradiation of point clouds in the sunshine and on vertical surfaces (i.e., façades) is computed as $u_v = u \cos(e) \sin(z^*)$, where $z^*$ is the difference between the azimuth of

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<td>$S$</td>
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<td>23</td>
<td>$R$</td>
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<tr>
<td>12</td>
<td>$e$</td>
<td>elevation</td>
<td>24</td>
<td>$R$</td>
<td>all the solar radiations</td>
</tr>
</tbody>
</table>

Fig. 2. The method used in the multiple reflection model. (a) A 3D shadow polygon $g_r$ made by a convex rooftop. (b) A 3D shadow polygon $g_f$ made by a concave rooftop. (c) A 3D shadow polygon $g_g$ interrupted by the other building. (d) Reflective radiations on the façade and ground.

Fig. 3. Shadows of buildings visualized in 2D. Each shadow is represented by a 3D polygon, where space below the polygon is in the shadow. Blue lines and red lines are intersection lines where 3D polygons intersect with the buildings on the façades and rooftops, respectively. Concave and nested polygons are also marked with the orange circles. (For interpretation of the references to colour in this figure legend, the reader is referred to the Web version of this article.)
2.5. Multiple reflections of solar radiation

A point \( p (p \in \mathcal{P}) \) is assigned to solar irradiation larger than 0 means that \( p \) has already received direct solar radiation \( sr \). The \( sr \), having arrived at the exterior surface of the urban area, will reflect and become reflective radiation denoted as \( rr \). More specifically, \( sr \) arriving at rooftops will reflect as \( rr \) and radiate upward with \( e \) and \( z \) unchanged. On the other hand, \( sr \) arriving at façades will also reflect as \( rr \) and further radiate downward with \( e \) unchanged while \( z \) is changed. When \( sr \) from the sky and \( rr \) from façades arrive at horizontal surfaces (i.e., rooftops or the ground), they will further transmit from the surfaces and directly radiate toward the sky or arrive at other façades with more reflections, and toward the sky ultimately.

It assumes that a complete set of direct solar radiation \( \mathcal{R} = \{sr\} \) has arrived at point clouds with solar irradiation larger than 0. Then, all the radiation in \( \mathcal{R} \) may have multiple reflection processes, as presented previously, which will incorporate into an iterative computational framework. In each reflection, \( sr \) will contribute an amount of the irradiation to the associated point clouds and maintain the remaining amount as \( rr \) according to a determined albedo \( \delta \). When \( sr \) arrives at a surface (either a rooftop, a façade, or the ground) and intersects the surface as a point \( o \), a circular buffer centered at \( o \) with a radius of \( r \) is created on the surface and is represented as \( \odot \). As a result, point clouds in \( \odot \) are associated with \( sr \), meaning that the irradiation will distribute to the point clouds evenly. Notably, during a reflection process, a point may receive a set of irradiations coming from different solar radiations, leading to an accumulation process. Fig. 4c demonstrates the result of the process of computing solar irradiation for urban envelopes after four rounds of reflections. Comparing with Fig. 4b, some areas accumulate fewer solar irradiation because of the albedo effect of solar radiation towards the sky, which takes away certain amount of the irradiation.

3. Empirical investigation

3.1. Study area

To obtain a knowledge of the effect of urban morphology on the solar capacity, the study selected ten cities across the world. The cities satisfy several criteria that (i) belonging to several representative climates, (ii) locating at a large range of latitudes (between 1.35° and 48.86°), (iii) reasonably flat so the effect of terrain variation on the distribution of solar radiation can be ignored, and (iv) possibly having different urban morphologies in the city. Since some cities have distinctly different urban morphologies, two study areas were also selected carefully for comparisons in each city, which are the downtown area with dense and relatively taller buildings (abbreviations appended by \( h \)) versus the residential area with sparse and relatively lower buildings (abbreviations appended by \( l \)) (Table 2). For example, TOh and TOl represent a typical downtown area and a typical residential area in Toronto, respectively.

3.2. Data collection

There is a challenge to obtain detailed building models covering all the large urban areas; alternatively, it assumes that rooftops are all flat. Based on footprints of buildings that are recorded as two-

**Table 2** Abbreviations of the study areas having averagely tall buildings verses averagely low buildings in ten cities.

<table>
<thead>
<tr>
<th>No.</th>
<th>Cities</th>
<th>Climate</th>
<th>Averagely tall (h)</th>
<th>Averagely low (l)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Athens</td>
<td>Mediterranean</td>
<td>ATH</td>
<td>ATI</td>
</tr>
<tr>
<td>2</td>
<td>Honolulu</td>
<td>semi-arid tropical</td>
<td>HOH</td>
<td>HOI</td>
</tr>
<tr>
<td>3</td>
<td>Hong Kong</td>
<td>subtropical</td>
<td>HK</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Lisbon</td>
<td>Mediterranean</td>
<td>LBH</td>
<td>LBI</td>
</tr>
<tr>
<td>5</td>
<td>Los Angeles</td>
<td>dry subtropical</td>
<td>LAH</td>
<td>LAI</td>
</tr>
<tr>
<td>6</td>
<td>Mandalay</td>
<td>subtropical</td>
<td></td>
<td>MA</td>
</tr>
<tr>
<td>7</td>
<td>New York</td>
<td>humid continental</td>
<td>NY</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Paris</td>
<td>continental</td>
<td>PAH</td>
<td>PAl</td>
</tr>
<tr>
<td>9</td>
<td>Singapore</td>
<td>tropical</td>
<td>SG</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Toronto</td>
<td>continental</td>
<td>TOH</td>
<td>TOI</td>
</tr>
</tbody>
</table>
dimensional polygons enriched with a height attribute, three-dimensional (3D), or more rigorously, 2.5D cities can be constructed. The building models in Hong Kong were obtained from the Lands Department of Hong Kong. Building models of five cities were obtained online, which include Singapore [43], New York [44], Los Angeles [45], Honolulu [46], and Toronto [47]. The model in Mandalay only has information on building levels [48], so the heights of buildings were estimated by assuming that each level was 3 m in height. High-resolution Digital Height Models (DHM) for Lisbon, Paris, and Athens were also obtained freely online [49], and the point clouds were constructed with the height information extracted from the DHM. Then, the average height was derived as the height of the building for points located in each building polygon derived from OpenStreetMap [50].

3.3. Computation

To obtain \( u \) from the sun at a specific \( t \) and \( l \), the Points Solar Radiation toolbox in the ArcGIS 10.3 software was used [51], the outputs of which demonstrate solar radiation on the horizontal surface. Since cloud cover is one of the most important factors that influence \( u \), the diffuse proportions and transmissivity of the atmosphere \( (\varepsilon_d, \varepsilon_t) \) were determined to calculate \( u \) precisely, from monthly cloud cover data collected over three years between 2015 and 2017 from World Weather Online [52]. To obtain the corresponding \((e, z)\) of solar radiation at \((t, l)\), this study also used an online tool named Sun Position [53].

Annual irradiations were computed based on the accumulation of hourly solar radiations over a year at a constant spatial resolution of 1 m, which created several millions of point clouds in each study area so that the computation could be extremely heavy. Therefore, the model was implemented in a relational spatial database management system (PostgreSQL 11) to achieve fast computation. Preliminary investigation suggested that more than 95% of the radiation ultimately reached the sky after four iterative reconstitutions because the largest irradiation of HK is smaller than the third quartile and the maximum. In the next four cities, ATl has the third quartile with a wide range between the third quartile and the maximum. Since PAh and PAl have similar cloud covers as HK according to historical weather data, the lower \( u \) of PAh and PAI can be caused by a lower annual direct irradiation at the higher latitude, even though they have higher solar capacities. Notably, NY (\(\alpha = 40.71\textdegree\)), ToH, and ToI (\(\alpha = 43.65\textdegree\)) have remarkably high \( u \) even though their latitudes are above 40°. Their large solar capacities (\(cap > 1\)) indicate that they can maintain higher solar irradiations than the same unit of flat area. This is probably because of the tall and dense buildings in the three areas, which is supported by clear evidence that façades in NY and ToI have a high proportion of the total irradiation.

A boxplot is presented to illustrate the numerical distributions of annual irradiations in ten cities (Fig. 9). MA, HOl, SG, and LAl have remarkably high solar capacities; NY, ToH, and ToI have remarkably high \( u \) even though their latitudes are above 40°. Their large solar capacities indicate that they can maintain higher solar irradiations than the same unit of flat area. This is probably because of the tall and dense buildings in the three areas, which is supported by clear evidence that façades in NY and ToI have a high proportion of the total irradiation.

The sensor was horizontal with the observed surface so that it could receive solar irradiation from all the directions without any obstruction. The sensor and the model obtained solar irradiations at the same location with a constant time interval, and each observation was for six to 7 h during a daytime. The observations have seven sites including three types of location, i.e., rooftop, the ground, and façade. The results show that relative errors decrease significantly in almost all the estimations when considering reflections (Fig. 7). As an exception, errors stay the same on the rooftops of Site 1 and Site 2 because the two rooftops have top heights in their surroundings so that no reflective radiation could contribute to the global irradiation. Specifically, the errors were lower than 0.1 in six observations out of nine when \(\delta = 0.40\), which achieves a satisfactory accuracy for the estimations. Thus, the empirical evaluation indicates that the model can accurately estimate solar irradiations in both spatial and temporal domains. Additionally, \(\delta = 0.40\) was used as an empirical parameter for the study.

4. Solar capacity of 3D cities

Annual irradiations in the ten cities were computed (Fig. 8). The Pearson correlation coefficient between the cosine of latitudes (\(cos(\alpha)\)) and annual irradiations (\(u_{all}\)) is \(R = 0.54\), which means that, as expected, a lower latitude tends to have larger annual irradiation. However, the correlation is only moderate, hinting at a possible role of urban morphology as a further determinant of a city’s solar capacity, an aspect that will be thoroughly analyzed in this study. Regarding correlations, areas with many high buildings can maintain larger irradiations, which becomes more noticeable when comparing two study areas in the same city and a significant exception occurs between ToH and ToI. Such a comparison also shows that SG (\(\alpha = 13.5\textdegree\)), MA (\(\alpha = 21.96\textdegree\)), and HOI (\(\alpha = 21.30\textdegree\)) have abundant \( u \). However, \( u \) in HK (\(\alpha = 22.39\textdegree\)) is rather low even though it still has a low latitude. This is principal because HK is mountainous, with a humid subtropical climate and frequent cloud cover, which significantly reduces solar radiation. Since PAh and PAI have similar cloud covers as HK according to historical weather data, the lower \( u \) of PAh and PAI can be caused by a lower annual direct irradiation at the higher latitude, even though they have higher solar capacities. Notably, NY (\(\alpha = 40.71\textdegree\)), ToH, and ToI (\(\alpha = 43.65\textdegree\)) have remarkably high \( u \) even though their latitudes are above 40°. Their large solar capacities indicate that they can maintain higher solar irradiations than the same unit of flat area. This is probably because of the tall and dense buildings in the three areas, which is supported by clear evidence that façades in NY and ToI have a high proportion of the total irradiation.
quartile is very similar to the ranking of the latitude, even considering the effect of cloud cover. Second, study areas having low-average building heights always have a larger third quartile than high-average building heights in the same city.

5. Spatial distribution of solar irradiations

Solar irradiations that are both numerically high and spatially concentrated are always preferred for many purposes, such as

### Table 3
Albedos of materials commonly seen in cities [54,55].

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<thead>
<tr>
<th>No.</th>
<th>Material</th>
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<tr>
<td>1</td>
<td>wall brick</td>
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</tr>
<tr>
<td>2</td>
<td>gypsum</td>
<td>0.35</td>
</tr>
<tr>
<td>3</td>
<td>ground asphalt</td>
<td>0.10–0.30</td>
</tr>
<tr>
<td>4</td>
<td>glasses (in average)</td>
<td>0.6895</td>
</tr>
<tr>
<td>5</td>
<td>concrete</td>
<td>0.17–0.27</td>
</tr>
</tbody>
</table>

Fig. 5. Annual irradiation, as resulting after the process of multiple reflections, in the 16 study areas of ten cities is visualized, with a detailed spatial scale of 500m×500 m.

Fig. 6. 3D point clouds of the field test sites. The area (945 m × 775 m) contains 302 buildings with heights ranging between 2 and 94 m. Both horizontal and vertical resolutions of the 3D point clouds are 1 m, which thus generates more than 1.4 million point clouds in total.
installing PV modules in cities. The efficient use of solar irradiations is unlikely if the distribution of the irradiations is fragmented. To explicitly understand the usability of solar irradiations in urban areas, spatial clusters of the irradiations on demand have been investigated in three partitions. Overall, study areas with lower buildings can provide more ground area $p(a)$ of given street blocks $p(b)$ to satisfy the solar irradiation demand (Fig. 10), where street blocks are spatial contiguous and homogeneous grid cells in resolution of 50 m. For example, in H01, at least 70% of the area ($p(a) = 0.7$) in 40% of the street blocks ($p(b) = 0.4$) can meet the criterion

\[ p(a) = 0.7, p(b) = 0.4 \]

Fig. 7. Relative errors between the observed and estimated solar irradiations. The bars marked with null in the x-axis were derived from ArcGIS software without the functionality of multiple reflections; the remaining bars represent the estimations that account for reflections with different albedo values, ranging from 0.25 to 0.40.

Fig. 8. Annual irradiations in 16 the study areas of ten cities across the world. The yellow bar presents the total amount of irradiations maintained by urban surfaces together with their partition into ground, rooftop, and façade components. The blue line shows the solar capacities for the 16 study areas. (For interpretation of the references to colour in this figure legend, the reader is referred to the Web version of this article.)
where solar irradiations are equal to or higher than 50% of the maximum irradiation $u_m$ (Fig. 10c). However, $p(a)$ is correspondingly only 0.4 in HOh. The figure also shows that HK and SC can maintain $p(a)$ effectively with the increase of the demand from 0.25$u_m$, 0.50$u_m$, to 0.75$u_m$ (Fig. 10a,10c,10e). This ability to maintain high values of $p(a)$ likely comes from their locations in low latitudes so that solar radiation at high elevations has a stronger ability to reach the ground. In contrast, $p(a)$ of the other cities drops off significantly when the demand increases. NY and ATl specifically have the lowest $p(a)$ for a given $p(b)$, since the densities of buildings in the two study areas are high and the elevations of solar radiation are small.

Similar to the ground, rooftops in study areas with lower average building heights have higher usability of solar irradiations if other factors remain constant (Fig. 11). For instance, $p(a)$ in NY, SG, and HK is lower than in HOI and LAl at a given $p(r)$ (Fig. 11a,11c). This phenomenon becomes even more apparent for two study areas in the same city, such as HOI versus HOh (Fig. 11a,11c,11e) and TOl versus TOh (Fig. 11b,11d,11f). Additionally, with the increase of $p(r)$ when the demand equals to 0.25$u_m$ and 0.50$u_m$, most curves in Fig. 11 decrease dramatically initially and then have a long and slight decrease followed by another dramatic decrease at the end. This indicates that large solar irradiations can be utilized on most rooftops, but the useable areas of large irradiations in these rooftops are limited. Besides, a small proportion of the area $p(a)$ can still utilize the highest demand of solar irradiations in SG, HOh, and HK in a small proportion of rooftops $p(r)$ (Fig. 11e). These rooftops are expected to be on high-rise buildings so that they are not affected by shadows most of the time. Specifically, almost all the rooftops in HOI allow 20% of the area to utilize the highest demand at 0.75$u_m$, since HOI is an area with low and sparse buildings, and solar radiation would thus easily arrive at most of the rooftops year-round.

In addition to the ground and rooftops, significant proportions of the area of some façades can be utilized effectively. More specifically, $p(a)$ can be maintained at 1.0 in most of the curves followed by a sharp decrease to the bottom with the increase of $p(v)$ (Fig. 12), which means that façades can usually allow either maximum or minimum usability. This phenomenon is evident for study areas such as NY and HK, which have high-rise buildings on average (Fig. 12a,12c,12e). Other study areas in the same city follow the same pattern and do not have distinct differences except for HOh and HOI (Fig. 12a,12b). The sharply decreasing trend without gradual changes is probably due to the orientations of façades, so façades with direct solar radiation can obtain maximum irradiations while façades continuously in shadow cannot obtain sufficient solar radiation even though multiple reflections of solar radiation may still contribute, such as in NY and HK.

All spaces including rooftops and the ground are scarce in urban areas, and utilizing façades of buildings to collect solar irradiations may be an effective strategy. Thus, a detailed investigation has been made of façades in the height domain. Overall, the average irradiation $v(u)$ increases while the proportion of the total irradiation $p(u)$ decreases gradually with the increase in the relative height $p(h)$ for all 16 study areas (Fig. 13). The optimal location to utilize solar energy is the height at which large solar irradiations of individual point clouds meet large amounts of the total irradiation. For example, $p(h) = 0.5$ may be an optimal height for PAI where $p(u)$ reaches the peak and $v(u)$ is relatively high. Notably, some of the blue curves have a slight decrease, which makes a saddle shape when it approaches the largest height. This unique pattern occurs in many study areas such as LAl, ATl, NY, and PAI. In NY for example, $v(u)$ continuously meets the first inflection point when $p(h) = 0.5$, a saddle point at $p(h) = 0.6$, and the second inflection point when $p(h) = 0.7$. This may be because several buildings have already reached their largest heights when $p(h) = 0.5$, causing several large irradiations to make a large $v(u)$. A deficiency in the number of buildings occurs and irradiations decreases when $p(h) > 0.5$, leading to a decrease in $v(u)$ until $p(h) = 0.6$. Furthermore, irradiations become larger when the remaining buildings still exist with an increase of $p(h) \in (0.6, 0.7)$. Additionally, some of the red curves have conspicuous irradiation accumulations at near-ground façades. Taking again NY for illustration, irradiations concentrating at $p(u) = 0.1$ may have two reasons for their existence. First, large areas at $p(u) = 0.1$ have already obtained considerable amounts of the total irradiation from direct solar radiation. Second, reflected radiation from the ground and other façades provides further
contributions that make \( p(u) \) higher than in an area with smaller \( p(h) \). This can also be seen intuitively by 3D visualizations of point clouds (Fig. 5).

Next, the study looks at the azimuthal domain and investigates the influence of building orientation on the distribution of irradianations on façades. The analysis demonstrates that the distribution of the irradiation varies when aggregating the total solar irradiation on façades in eight directions (Fig. 14). For example, solar irradiation in NY has even distributions from three directions, SE, SW, and NW, while it is insignificant from NE and is nearly zero for S and W. This coincides with orientations of buildings given that buildings in the NY study area are regularly oriented in a south direction. A similar phenomenon occurs in several other study areas, such as MA, ToH, and ToL. However, building orientations are inhomogeneous in some other study areas such as Lbh and Lbl, meaning the distribution occurs in almost all directions. Therefore, orientations of buildings can significantly affect the directional distributions of solar irradianations.

6. Effects of urban morphology

This principle, and the previously made observation about the moderate correlation of latitude with the annual irradiation, motivates the investigation of how urban morphology affects solar energy potential, and what urban morphology is most desirable to effectively utilize solar energy. Eight indices have been proposed (Table 4) to describe different aspects of urban morphology and then correlate with the solar capacity (Fig. 15). Compared with \( u_a \) and \( \cos(a) \), which have a positive correlation with \( R = 0.54, p = 0.032 \) as mentioned in Section 4, \( \text{cap} \) and \( \cos(a) \) have obtained moderately negative correlations given \( R = -0.47 \), thereby suggesting that latitudes have a significant but not
conclusive impact on solar capacity (Fig. 15a). This reveals that cities at higher latitudes tend to have a relatively larger cap even though their ua is generally smaller. If all the conditions are the same, an area in a high latitude tends to have large shadows and a small direct irradiation coming from the atmosphere. However, a high latitude will also make positive impacts on maintaining the total irradiation when considering reflective irradiations. First, the solar-face area receives all the irradiations that thus makes shadows in the dark-side area so that shadows cannot determine the capability of maintaining the irradiation. Second, a higher latitude means a lower annual elevation of the solar radiation, which creates more reflections between facades and leads to more accumulation of the total irradiations. This impact becomes more significant in high-density areas.

Orientations of buildings may influence the solar capacity because statistics have suggested that solar irradiations on façades are different in different study areas (Fig. 14). Therefore, the correlation between cap and the density of solar-face area d(slr) has been calculated. The analysis shows that cap and d(slr) have a somewhat strong and positive correlation with R = 0.60 (Fig. 15b), which means that the orientation of buildings can significantly influence the solar capacity of cities. Surprisingly, TOh has a smaller cap but a much larger d(slr) compared with TOl, presenting a negative correlation for the two study areas in the same city. Visualizing all the irradiations in 3D point clouds indicates that many high-rise buildings are located in the south and east part of TOh; therefore, a large amount of incoming solar radiation arrives at the buildings, and then reflects to the sky immediately, while the majority of buildings in the north and western parts are under the shadow, which results in lower irradiations being maintained by the buildings. Thus, cap decreases for TOh even though d(slr) is larger. The two findings clearly illustrate the importance of
orientational planning of buildings and associated locational height control to maximize the solar capacity.

Even when the façades in $d(slr)$ satisfy the condition $|d| < \frac{\pi}{2}$, some can still be blocked by other buildings and are essentially in shadow, such as in TOh. Therefore, the density of brightness areas, denoted by $d(bgt)$, was considered while disregarding the orientation of buildings. Since boundaries between shadows and brightness are ambiguous in many urban areas, a set of values of $x$ should be tested empirically to determine the brightness points. Interestingly, $cap$ and $d(bgt)$ are in a very strong and positive correlation of $R = 0.84$ when $x = 0.1$ (Fig. 15c). The analysis clearly shows that a larger $d(bgt)$ leads to a larger $cap$ for any two study areas in the same city without any exception. The analysis also finds that some irradiation accumulations are not located in the solar-face area, thereby suggesting a significant contribution of multiple reflections from other buildings. Therefore, $d(bgt)$ appears more significant than $d(slr)$ in determining solar capacity.

One of the most apparent differences between study areas is the majority of high buildings in some cities versus a majority of low buildings in others, which creates distinct differences in urban morphology and may have certain effects on solar capacity. Therefore, the heights of all point clouds on horizontal surfaces of rooftops and ground, denoted by $h$, were investigated. The $cap$ and the average height of the points $v(h)$ showed a somewhat strong and positive correlation of $R = 0.78$ (Fig. 15d), meaning that a study area with overwhelmingly high buildings tends to have a relatively larger solar capacity. This may be mainly because high buildings can maintain large amounts of irradiation on façades (Fig. 12). However, study areas with the same $v(h)$ may still have different solar capacities since the difference between each height and $v(h)$ can be different. Thus, the concept of the roughness of urban envelopes is introduced to describe the topographic relief in study areas, which is represented by $r(h)$. The analysis shows that $cap$ and $r(h)$ also obtained a somewhat strong and positive correlation of $R = 0.75$ (Fig. 15e), which means that large erratic fluctuations in topography would lead to a large solar capacity. This is because the heights of buildings varying randomly allow solar radiation to pass through various parts of the urban area so that large amounts of irradiations can be maintained and accumulated through multiple reflections between buildings. For example, given two equal areas in the same location and having the same building footprints, one has the same height for all buildings and $r(h) = 0$, while the other has various

Fig. 12. A rate of $p(a)$ area in a rate of $p(v)$ façades has solar irradiations equal or higher than $x$ $(x = \{0.25, 0.50, 0.75\})$ rate of the maximum irradiation $(um)$ on the façade. For two study areas in the same city, $um$ equals to the larger value of the two maximums on the façade for fair comparison. (a), (c), and (e) Six study areas with three demand rates. (b), (d), and (f) Four study areas with three demand rates.
heights of the buildings and \( r(h) > 0 \). All the radiation arriving at the rooftops will reflect toward the sky without any further accumulation of irradiations when \( r(h) = 0 \). In contrast, when \( r(h) > 0 \) radiation arriving at rooftops and/or façades can reflect onto façades and/or the ground with the accumulation of irradiations simultaneously.

Large areas of urban envelopes, including ground, rooftops, and façades, will likely receive and maintain solar radiation. Motivated by this observation, the density of urban envelopes was computed and denoted as \( d(\text{sur}) \). Surprisingly, \( \text{cap} \) has a very strong and positive correlation with \( d(\text{sur}) \) of \( R = 0.85 \) (Fig. 15f), which demonstrates that a large area of urban envelopes can promote the maintenance of irradiations from direct and reflected radiation. Notably, \( d(\text{sur}) \) is determined by the area of façades since the total area of the ground and rooftops is deterministic and is equal to the flat area where it locates. Furthermore, the decomposition of the façade areas can be indicated by \( v(h) \) (Fig. 15d) and \( v(l) \). Since \( \text{cap} \) and \( v(l) \) have also obtained a somewhat strong and positive correlation of \( R = 0.66 \) (Fig. 15g), this suggests that the correlation of \( R = 0.85 \) between \( \text{cap} \) and \( d(\text{sur}) \) can be explained by the combined effect of \( v(h) \) and \( v(l) \). However, \( \text{cap} \) and \( d(\text{plg}) \) only show a moderately positive correlation of \( R = 0.45 \) (Fig. 15h). This means that a large \( d(\text{plg}) \) can lead to a large \( \text{cap} \). Notably, study areas such as PAl, ATh, and ATl have a relatively high \( d(\text{plg}) \) but a small \( \text{cap} \). The reason for this result is that \( d(\text{plg}) \geq 0.47 \) calculated for the three study areas indicates that there

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**Fig. 13.** Distribution of solar irradiances along heights of façades. The average irradiation \( v(u) \) is the blue curve and corresponds to the left y-axis. Proportion of the total irradiation \( p(u) \) is the red curve and corresponds to the right y-axis, both of which are against heights of point clouds on façades relative to the maximum height denoted by \( p(h) \) in the x-axis. (For interpretation of the references to colour in this figure legend, the reader is referred to the Web version of this article.)

**Fig. 14.** Distribution of proportion of the total irradiation on façades in eight directions.
is not much space between buildings. Hence, only a small proportion of the solar radiation can arrive at the top of façades, and most of the solar radiation reflects to the sky immediately after it arrives at the rooftops and on the ground. This phenomenon becomes even more evident for the three study areas since the heights of these buildings are rather homogeneous, i.e., $r(h)$ is small.

### Table 4

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<th>No.</th>
<th>Index</th>
<th>Meaning</th>
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<tr>
<td>1</td>
<td>$\cos(a)$</td>
<td>Cosine of the latitude.</td>
</tr>
<tr>
<td>2</td>
<td>$d_{slr}$ = $\frac{\sum (\cos(d), a_i)}{s_a} (</td>
<td>d_i</td>
</tr>
<tr>
<td>3</td>
<td>$d_{bgt}$ = $\frac{\sum (\cos(b), a_i)}{s_a} (u_{bgt} \geq x_{\text{um}})$</td>
<td>The density of brightness areas. The total area of point clouds in brightness areas is divided by the entire study area, where the point clouds equal to or higher than $x$ proportion of the maximum annual irradiation are viewed as the annual brightness area.</td>
</tr>
<tr>
<td>4</td>
<td>$v(h)$</td>
<td>The average of the building heights.</td>
</tr>
<tr>
<td>5</td>
<td>$r(h)$</td>
<td>The standard deviation of the heights on horizontal surfaces.</td>
</tr>
<tr>
<td>6</td>
<td>$d(sur)$</td>
<td>Total area of urban envelopes divided by the corresponding flat area.</td>
</tr>
<tr>
<td>7</td>
<td>$v(l)$</td>
<td>The average perimeter of all the footprints of buildings.</td>
</tr>
<tr>
<td>8</td>
<td>$d(plg)$</td>
<td>The density of all the footprint areas.</td>
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</table>

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**Fig. 15.** Pearson correlation coefficient ($R$) between eight indices and the solar capacity.
7. Discussion

The model supports rooftops in different geometrical shapes, e.g., a rooftop made by two slanting polygons, while the study assumes that all rooftops are flat due to lacking a high level-of-detail dataset. Nevertheless, this hypothesis should not cause a notably adverse effect because a vast majority of reflective radiations from the rooftops approach the sky immediately that will not make an influence on estimating solar irradiation on urban envelopes. Even though the albedo used in the study is a constant value derived from an empirical investigation, it will not make any effect on solar capacity and the correlation analysis with urban morphology because solar capacity is a relative index essentially that counts the percentage of the annual irradiation. More accurate albedo for each city can obtain from recognizing building materials that come from street-view images, and machine learning could be an effective way to process large amounts of the images. The model can also be improved to account for the influence of terrain variation and vegetation, which in some urban areas is highly relevant.

The specular reflection rule is used to approximate all the reflections to control the computational cost. Even a small area may contain millions of point clouds, so the intersections between all the reflective radiations and all the urban envelopes would generate thousands of millions of associations as an intermediate result, which would further burden an already cumbersome computation process. Thus, the diffuse reflection on concrete surfaces of the urban envelopes is simplified to maintain reasonable computational efficiency.

Having studied the structure of 3D solar cities, people can determine the optimal locations for utilizing solar energy based on each area’s specific needs. For instance, installations of PV modules should be at surfaces directly illuminated by the sun where irradiations are both quantitatively high and spatially concentrated, vegetation for greenery purposes shall be in moderate sunshine, and thermal comfort routing planned for pedestrians should be at surfaces directly illuminated by the sun where irradiation between 3D cities so that shared electric scooters can get charging during the riding gaps, which could reduce extensive manpower for charging and repositioning trips significantly and thus improves the availability of the scooters.

8. Conclusion

This study proposed a solution to construct a 3D solar city, which considers not only direct and diffuse radiation from the atmosphere but also incorporates reflective radiation between 3D urban envelopes. Accuracy assessment obtained smaller relative errors, suggesting that the reflective module approximates solar distribution more accurately in an urban environment since high albedo in cities may alter the distribution significantly. Correlation analysis between solar capacities and indices of urban morphology is strong and positive overall even though latitude and clouds influence total solar irradiation, which suggests that urban morphology has a substantial impact on solar capacity, which thus makes a disruptive change on annual irradiation ultimately. It also suggests that, in the same condition (latitude and clouds), a city tends to have a larger solar capacity when it has a higher density of tall buildings and an erratic fluctuation of the building heights. The proposed indices of urban morphology can support local governments in strategizing urban development, and the robust computational framework of the model can be applied to provide adequate decision-making support for initiatives.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

CRediT authorship contribution statement

Rui Zhu: Conceptualization, Methodology, Software, Validation, Formal analysis, Resources, Visualization, Writing - original draft, Writing - review & editing.
Man Sing Wong: Conceptualization, Supervision, Writing - review & editing.
Linlin You: Resources, Formal analysis, Writing - review & editing.
Paolo Santi: Conceptualization, Supervision, Writing - review & editing.
Janet Nichol: Writing - review & editing.
Hung Chak Ho: Writing - review & editing.
Lin Lu: Validation.
Carlo Ratti: Conceptualization, Supervision.

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Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.renener.2020.02.050.

References